Quantum Computing for EO Mission Planning

Talk for EO Workshop at QIM-V Conference
Earth Observation Trends and Computational Challenges

**EO market transformed by a number of New Space initiatives**
- Affordable access to Space (thanks to reusable launch systems)
- Low-cost recurring platforms and instruments (based on COTS)
- Smart multi-mission ground systems (leveraging AI & Big Data technologies)

**Computational challenges for future EO constellations**
- Optimization at design phase
  - Constellation pattern / orbital configuration
  - Ground segment sizing
  - Communication links / network topology
  - Reliability, Availability, Maintainability, Safety across system lifecycle
- Decision making during operations
  - Mission planning
  - Resource Management
  - Operations Scheduling
  - Data flow and network routing
EO Mission Planning
EO Mission Planning Problem Statement

**Mission Planning**: Must determine an optimal acquisition plan for an Earth Observation satellite

**Input data**
- \( R \) is the set of acquisition requests, \( I_r \) is the set of imaging attempts for request \( r \in R \).
- \( \forall r \in R, \forall i \in I_r, w_{r,i} \) is the score of the imaging attempt
- \( \forall r \in R, \forall i \in I_r, t^r_i \) is the start time of the imaging attempt

**Decision variable**
- \( x_{r,i} \) is the binary variable indicating whether the candidate attempt \( i \) is selected in the plan
  - The number of binary variables is \( N_{\text{variable}} = \sum_r |I_r| \)

**Constraints**
- Maximally one assigned attempt \( i \) per request \( r \): \( \forall r \in R, \sum_{i \in I_r} x_{r,i} \leq 1 \)
- Some consecutive imaging attempts are not possible:
  - \( F_{r_1,r_2} = \{(i,j) \in (I_{r_1},I_{r_2})| t^r_{i_1} \leq t^r_{j_2} \& \& t^r_{j_2} < t^r_{i_1} + T_{r_1 \to r_2, \text{acquisition}} + T_{r_1 \to r_2, \text{maneuver}} \} \)
  - \( \forall (r_1,r_2) \in R^2, \text{with } r_1 \neq r_2, \forall (i,j) \in F_{r_1,r_2}: x_{r_1,i} \cdot x_{r_2,j} = 0 \)

**Objective**: Total score of the schedule
- Minimize \( C = -\sum_r \sum_{i \in I_r} w_{r,i} x_{r,i} \)
Quantum Computing in a Nutshell

**Superposition Principle**
- A qubit can be seen as a superposition of two basis vectors:
  \[ |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \]
- A n-qubit register represents a \(2^n\)-dimensional vector space, allowing for exponentially greater information processing.

**Classical vs Quantum Computing**

<table>
<thead>
<tr>
<th>Steps</th>
<th>Classical Computation</th>
<th>Quantum Computation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Initial State</td>
<td>A single length-N binary string e.g. 00</td>
<td>Superposition of length-N binary strings e.g. (\frac{</td>
</tr>
<tr>
<td>2 Apply Operations</td>
<td>Logic gates e.g. 00 → 01</td>
<td>Quantum logic gates e.g. (\frac{</td>
</tr>
<tr>
<td>3 Read Out</td>
<td>Read out e.g. READ(01) → OBSERVED: 01</td>
<td>Measure e.g. MEASURE (\frac{</td>
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\[2^n \times 2^n\] Unitary operators

\[\sum_{i=1}^{n} |x_i|^2 = 1\]
Quantum Computing in a Nutshell

Quantum Annealing Computer (D-Wave)
- Not a general purpose quantum computer, but uses quantum properties to solve discrete optimization problems
- Natural evolution of quantum-mechanical system (using quantum tunnelling) towards a ground state minimizing its energy

Circuit Model Quantum Computer (IBM, Google)
- Quantum circuits are composed of elementary gates and operate on qubits
- QC equivalent to classical boolean feed-forward networks, except they are reversible (i.e. quantum circuits can be evaluated in both directions)
Quantum Computing in a Nutshell

Quantum Computing for Combinatorial Optimization

Quantum Annealing
• Solves Quadratic Unconstrained Binary Optimization (QUBO) problems
  i.e. minimize \( H(\{x_i\}) = \sum_i Q_{ii}x_i + \sum_{i<j} Q_{ij}x_ix_j \), where \( x \in \{0, 1\}^N \)
• Requires to formulate your discrete optimization problem as a QUBO
• Remains an approximate optimization technique

Quantum Approximate Optimization Algorithm
• Finds the bit string that maximizes a set of logical clauses
  \( C(\{x_i\}) = \sum_{\alpha} C_\alpha(\{x_i\}) \)
• Starts from an initial uniformly distributed quantum superposition of all possible states and uses a quantum circuit to iteratively apply unitary operators, yielding an approximate solution to the problem
QUBO Problem Formulation

Quadratic Unconstrained Binary Optimization (QUBO) formulation

- QUBO: \( \min q(x) = x^T Q x = \sum_{j=1}^n Q_{j,j} x_j + \sum_{j,k=1}^n Q_{j,k} x_j x_k \) with \( Q \) an upper-triangular quadratic matrix

- Constraint equations in a quadratic form:
  - (1) : Max one imaging attempt per request : \( C_u = \sum_{i,j \in I_r, i < j} \min\{w_{r,i}, w_{r,j}\} x_{r,i} x_{r,j} \)
  - (2) : Non feasible maneuver \( C_t = \sum_{r_1,r_2} \sum_{i,j \in F_{r_1,r_2}} \min\{w_{r_1,i}, w_{r_2,j}\} x_{r_1,i} x_{r_2,j} \)

- Constraints are taken into account in the QUBO formulation to minimize
  - \( q = C + \lambda_u C_u + \lambda_t C_t \)
  - With :
    - \( C = -\sum_{i \in I_p} w_{r,i} x_{r,i} \) is the objective function in the original problem
    - \( \lambda_u, \lambda_t \) are penalty weights

- Choice of the penalty weights
  - Sufficiently large enough such that \( \hat{x} = \arg \min_x q(x) \) verifies our constraints, i.e. \( C_u(\hat{x}) = 0 \) and \( C_t(\hat{x}) = 0 \)
  - We can demonstrate that any choice of penalty weight values such that both \( \lambda_u > 1 \) and \( \lambda_t > 1 \) gives valid solutions
QUBO Problem Formulation

Ising formulation and embedding

- Ising-Model is required to map to D-Wave machine
  - \( I(s) = \sum_i h_i s_i + \sum_{ij} J_{ij} s_i s_j, \quad s_i \in \{-1, 1\} \)
  - \( x_i = (s_i + 1) / 2, \quad x_i \in \{0, 1\} \)

- Due to D-Wave architecture (chimera graph), a physical qubit is not connected to every other qubit
  - **Embedding** is the process of linking physical qubits together to virtually enhance connectivity
  - In our case, problem instances need to stay below **80 logical qubits** to be embeddable on the D-Wave machine
Mission Planning Simulation

Mission Planning problem instances
- Generated thanks to Airbus DS Mission Simulator (TEAM)
- Reduced instances with a small number of requests and a coarse access discretization compared to real operations
- Different scenarios are considered to generate multiple instances, enabling sensitive analysis and statistics on average performance
- Main parameters
  - Number of acquisition requests
  - Access discretization step
  - Latitude range for Area of Interest
    - Drives the number of decision variables
    - Drives the “NP-hardness” of the planning problem

Examples:
- Nb of requests = 11
  - Discretization step = 12s
  - Latitude range = 10°
- Nb of requests = 12
  - Discretization step = 16s
  - Latitude range = 1°
Evaluation on Classical Hardware

Two classical algorithms have been considered

- An exact MIP solver, showing an exponential runtime
- A greedy algorithm (similar to operational software), showing a linear runtime (at least for small instances)
Evaluation on D-Wave Quantum Annealer

Performance Assessment methodology

- A number of annealing runs is configured
- A success probability is derived: \( p = \frac{\text{# exact solutions}}{\text{# annealing runs}} \) (probability to yield an optimal solution)
- Assuming independence between runs, a time-to-solution with 99% chance of optimality can be expressed as
  \[ T_{99} = \frac{\ln(1-0.99)}{\ln(1-p)} T_{\text{Annealing}} \]

D-Wave Configuration

- Number of annealing runs (10000)
- Annealing time (20 \( \mu s \))
- Choice of intra-logical qubit coupling \( J_F \)
- Embeddings: using all 5 D-Wave heuristic embeddings
- Unembedding strategy: majority vote
Classical / Quantum Computing Benchmark

**Bottom Line**

- As expected, no quantum speedup could be obtained through these experiments.
- Both classic and quantum execution time increase exponentially with the number of binary variables.
- In the near term, quantum annealers may nevertheless outperform classical computers in solving discrete optimization problems.
Conclusion and Perspectives

Technical achievements

• Classical vs Quantum benchmark for the exact optimization of a broad range of (small) satellite mission planning problems
• Although no quantum speedup was obtained, the run-time performance on D-Wave Q Annealer (at its current scale) is very promising

Perspectives

• Hybrid Classical-Quantum approach for Satellite Mission Planning
  • Decomposition methods to optimize satellite mission planning on real-size problems
  • Iterative schemes and metaheuristics mixing Classical / Quantum Annealing

• Quantum Approximate Optimization Algorithm for Satellite Mission Planning
Airbus CTO launched the Quantum Computing Challenge to provide the Quantum Computing community with industrial related problems, enabling the assessment of the potential of Quantum Computing on Airbus Commercial applications.

5 Problem Statements
- Aircraft Climb Optimization
- Computational Fluid Dynamics
- Quantum Neural Networks for Solving Partial Differential Equations
- Wingbox Design Optimization
- Aircraft Loading Optimization

Thank you